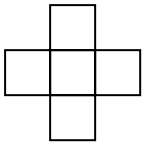
UVAMT 2025 - Team Round

T1. On a table, some coins are placed in a row, each showing heads or tails, such that:

- Exactly 3 coins are showing tails,
- Each tail is adjacent to two heads, and
- Each head is adjacent to exactly one other head.

How many coins must there be on the table in total?

T2. Remove four 1x1 squares from the corners of a 3x3 square to obtain the following figure:



It turns out that this figure fits in a square smaller than 3x3! What's the minimum side length of a square that this figure can fit in?

T3. Mikhail wants to fill in the empty cells in this grid with integers, such that every row, column, and 3-cell diagonal sums to the same value v. Find v.

		4
3		
	5	8

T4. How many 4-digit numbers have exactly 3 odd digits?

T5. Suppose that x is an integer such that both x and $x+45$ are perfect squares. Determine the sum of all possible values of x.

T6. Thirty students are in a school that offers three courses. Each course is taken by exactly 21 students. Find the maximum possible number of students taking exactly one course.

T7. Let a positive integer n be *satisfying* if there exists a geometric progression $a,\ b,\ c$ of positive integers $a < b < c$ with $a+b+c = n$. Find the smallest n such that both n and $n+2$ are satisfying.

T8. Find the sum of all real numbers $a < 2025$ such that $x\lfloor x \rfloor = a$ has at least two solutions.

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T9. Utkarsh and Vincent are moving to UVAMT grounds, which are shaped like a perfect circle. Utkarsh is at the center of grounds, and Vincent is at the edge. They pick a random spot uniformly within grounds to meet up. What is the probability that Utkarsh is closer to this spot than Vincent?

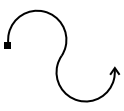
T10. How many non-constant polynomials with integer coefficients pass through the point $(-2,\ 27)$ and have all positive integer roots?

T11. Two straight roads intersect at an angle θ. Utkarsh and Mikhail are both 1 mile away from each of the roads, but Utkarsh is twice as far from their intersection as Mikhail is. Find $\cos\theta$. Note: You may assume that the roads are infinitely long in both directions.

T12. Eight vertices are placed in 3D space to form a unit cube. Utkarsh chooses four distinct vertices randomly out of these eight. What is the expected volume of the tetrahedron formed by these four vertices? (Note: if all four vertices lie on the same plane, the volume is said to be 0 by convention.)

T13. Utkarsh selects an integer between 1 and n inclusive, and Vincent makes a series of guesses to try to determine Utkarsh's number. When he makes a guess, Utkarsh tells him whether it was greater than, less than, or equal to the correct number. If it was greater, Utkarsh gets 3 points; if it was less, Utkarsh gets 1 point; if it was correct, the game ends. Find the greatest n such that Vincent can ensure Utkarsh scores at most 10 points.

T14. You're driving a go-kart without gas or brakes - just a steering wheel. The go-kart travels in arcs of radius 1, and you can instantaneously change the direction of its arc between clockwise and counterclockwise, but this is all the control you have. You're currently at point A traveling due north. What's the minimum distance you must travel to return to point A traveling due south? (An example of the go-kart's movement pattern is given below.)



T15. For positive integer x, let $f(x) = \begin{cases} x/2 & (x \text{ even}) \\ x+1 & (x \text{ odd}) \end{cases}$. Let $g(x)$ be the minimum k such that $\underbrace{f(f(\ldots f(x)))}_{k} = 1$. Compute $\sum_{x=2}^{2^{2025}} g(x) \bmod 999$.