UVAMT 2026 Individual Round

I1. How many ways are there to fill in the boxes with two-digit positive integers such that the equation below is true?

$$100 - \square = \square$$

I2. Some cards are placed on a table, all either face-up or face-down. Currently, there are exactly 50% more face-up cards than face-down cards. What percent of the cards need to be flipped over so that there are an equal number of face-up and face-down cards?

I3. Mikhail has two types of coins: some 10-cent coins and some 3-cent coins. It is known that he has at least one of each type, and his coins' total value is 50 cents. How many coins does he have in total?

I4. A scientist has recorded n temperature readings. A collection of readings is considered near freezing if their average temperature is between -1° and 1°. Suppose that:

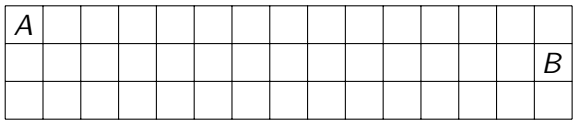
- If she adds an additional reading of 3°, the resulting collection is near freezing, and
- If instead she adds an additional reading of 15°, the resulting collection is still near freezing.

What is the smallest possible number of readings she could have started with?

Clarification: The range -1° to 1° is inclusive.

I5. Utkarsh starts out 1 mile away from his house, but he doesn't know which direction it's in. He decides to pick a random direction, and walks 1 mile. What is the probability that he is now closer to his house than before? Express your answer as a probability between 0 and 1.

I6. A knight starts at position A on a 3×15 grid. In a single move, a knight moves in an L-shape: 2 squares in one direction and 1 square in a perpendicular direction. How many ways are there for it to reach position B using exactly 7 moves?



I7. Suppose six distinct positive integers are selected such that all of them are factors of 210, but no two of them are divisible by each other. What is the sum of these six integers?

I8. A group of 50 students are asked to rank how much they enjoy the following fruits: apples, bananas, cantaloupe. It turns out that the majority (greater than half) of students prefer bananas over apples, and the majority of students prefer cantaloupe over bananas. What is the maximum possible number of students who prefer apples over cantaloupe?

I9. How many real numbers x satisfy the equation

$$x^2 + \tan^2 x = 100,$$

where the argument to tan is given in radians?

I10. Let S be a regular hexagon. Suppose that any point in S can be covered by a length-1 line segment that stays entirely within the hexagon. What is the minimum possible side length of S?

I11. Three racers are running laps on a circular racetrack. It takes them x, y, and z minutes respectively to complete one lap. They all start at point A. Suppose that at some time, all the runners are at the point diametrically opposite to A. For how many integer values $1 \leq x, y, z \leq 10$ is this possible?

Clarification: Solutions (x, y, z) are ordered and not necessarily distinct.

I12. An urn starts out with 2026 stones, labeled 1 through 2026. Every day, Vincent selects two stones at random from the urn, discards the stone with the higher label, and places the stone with the lower label back into the urn. He repeats this process until only two stones remain. What is the probability that the stone labeled 2 is still in the urn at the end?

I13. Find the sum of all integers n with $|n| \leq 2026$ for which there exist quadratics $P(x), Q(x), R(x)$ with integer coefficients such that

$$P(x)Q(x)R(x) = x^6 + n.$$

I14. Four cups are on a table, one full and the other three empty. At each step, two distinct cups are chosen uniformly at random, and the water among them is split evenly between them. On average, how many steps does it take until all four cups contain the same amount of water?

I15. Let a function $f(x)$ be *hypercubic* if it can be expressed in the form

$$f(x) = ((\cdots((x + a_1)^3 + a_2)^3 + \cdots)^3 + a_{n-1})^3 + a_n,$$

where $a_1, a_2, \ldots, a_n$ are integers, and n can be any positive integer. (The value of n may vary between hypercubic functions.) Suppose a set

$$S = \{f_1, f_2, \ldots, f_k\}$$

of hypercubic functions satisfies the following property: there are no two distinct functions $f_i, f_j \in S$ such that $f_i(m) - f_j(m)$ is divisible by 10 for all integers m. Find the maximum possible value of $|S|$.